

# Map Projections

## Cartographic Information Systems

Second Edition

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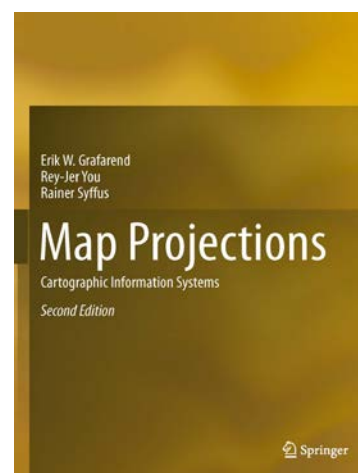
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## The Wonderful World of Map Projections

A map projection is a mathematical method for visual representations of points, curves or surface pieces on the Earth or other celestial bodies. This book covers general theories and applications of map projections. The main concern in this book is the mapping from a two-dimensional *Riemann* manifold, which has a positive-definite metric tensor, to another two-dimensional *Riemann* manifold, in particular mapping from the Earth surface as an ellipsoid-of-revolution or a sphere to a plane, a cone, or a cylinder in normal, oblique, and transverse aspects.

Based on *Gaussian Theorema Egregium*, mapping an ellipsoid-of-revolution or a sphere to a plane, a cone, or a cylinder is impossible without distortions or deformations. Depending on the purpose of the mapping or the requirements of distortion control, different map projections are systematically developed using *Differential Geometry* in this book. The distortion or deformation measures, which are the basic issues in map projections, are discussed in detail.

This book has 935 pages and is bound in two volumes. Its contents are mainly divided into three parts. The first part contains the first four chapters where offer foundational differential geometric material of map projections. They are concerned with general mapping theories from a *Riemann* manifold to another *Riemann* manifold. The *Cauchy* and *Green*



deformation tensors are presented and the eigenspace of the tensors which is important for distortion analysis and then for mapping types is studied. The criteria of optimal map projections for a specific purpose are presented and solved by variational calculus in this part. The second part of this book in Chapters 5-24 focuses on the mapping from an ellipsoid-of-revolution or a sphere to a plane, a cone, or a cylinder. Different distortions, namely conformal, equiareal, equidistant, perspective, and geodesic, are designed, derived, and analyzed for every specific map projection mentioned in this part. The mapping of torus, hyperboloid, paraboloid, onion shape, and clothoid for structures is also reviewed. The datum transformation is discussed in Chapters 21 and 24. In particular, the ten parameter conformal group  $C10(3)$  is introduced as a datum transformation method of the three-dimensional Euclidean space. The third part of this book is the 10 appendices where give useful supplements, such as mathematical laws and order and the inverse of a multivariate homogeneous polynomials. Elliptic integrals, *Korn-Lichtenstein* and *d'Alembert-Euler* equations are introduced. Special map projections, in particular mixed cylindrical, generalized *Mollweide*, generalized *Hammer*, polycylindrical map projections, are derived. Geodesics and *Gauss* surface normal coordinates in geometric and gravity spaces which may be especially interesting for geodesists and geophysicists are also reviewed. Besides, the *Newton* form of a geodesic in the *Maupertuis* gauge is presented and the concept of projective heights is introduced in the third part. In addition to the theories of map projections, applications and numerical examples are also given in this book.

This professional book basically describes map projections from differential geometric point of view and aims to provide a completely systematic and theoretic construction of map projections for the students and academics working on Cartography, GIS, Spatial Information, Computer Vision, Geodesy, Geophysics, and other related field.

by Rey-Jer You  
Tainan, 05.May 2015

# 地圖投影

## 製圖資訊系統

第二版

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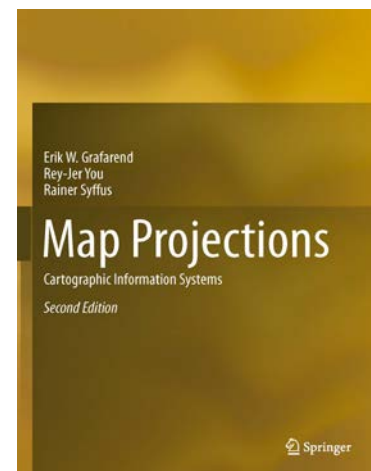
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## 地圖投影的奇妙世界

地圖投影是一種視覺表現地球或其他星體表面之點、線或面的數學方法。這本書涵蓋了地圖投影的一般理論和應用。在這本書中, 主要關注的是從一個具有正定度量張量的二維黎曼流形映射到另一個二維黎曼流形, 特別是從一個球面或旋轉橢球面映射到正常的, 斜的或橫向三個不同方位的平面、錐形、或(橢)圓柱形上。

基於高斯 *Theorema Egregium* 理論, 將球面或旋轉橢球面映射到平面、錐形、或橢圓柱形上是不可能沒有扭曲或變形。根據映射的目的或畸變控制的要求, 本書使用微分幾何以系統性地推演不同的地圖投影, 並對地圖投影中的基本扭曲或變形問題進行了詳細討論。

這本書共 935 頁, 裝訂成上、下兩冊, 內容主要分為三部分: 第一部分包含了前四章, 這四章提供了地圖投影的基本微分幾何材料, 並聚焦於從一個黎曼流形映射到另一個黎曼流形的一般理論。同時, 推演出 *Cauchy* 和 *Green* 變形張量和分析他們的特徵空間, 而這些特徵空間是扭曲或變形、以及地圖投影類型的重要參考依據。為特定目的之最佳地圖投影之規範被提出並應用變分法求解最佳化問題。這本書的第二部分為第 5-24 章, 這些章節側重於從球面或旋轉橢球面映射到平面、圓錐、或(橢)圓柱形上。根據不同的正形、等面積、等距, 透視和大地測量之特定扭曲或變形, 對地圖投影方法進行設計、推導、和分析。另外, 我們還複習了環面、雙曲面、拋物面、洋蔥形狀和克羅曲線梭的投影方法在特定結構物上的應用。基準轉換問題則在第 21 章和第 24 章討論, 並特



別介紹了 10 個參數的正形轉換羣  $C10(3)$  作為三度歐基理德空間的基準轉換方法。第三部分是 10 個附錄，提供有用的補充材料，如數學定律和次序關係，以及多元齊次多項式的逆轉換，且也對橢圓積分、*Korn-Lichtenstein* 和 *d'Alembert-Euler* 方程式進行了介紹。特殊的地圖投影，尤其是推演了混合的(橢)圓柱形，廣義 *Mollweide*，廣義 *Hammer*，*polycylindrical* 等地圖投影方法。同時，複習了測地線和高斯表面法線坐標於幾何和重力空間的應用，它們可能是大地測量學家和地球物理學家特別有趣的。此外，牛頓形式的 *Maupertuis* 測地線的呈現與投影高的概念也在第三部分介紹。除了地圖投影理論外，本書也在適當的地方給出地圖投影的應用和數值算例。

本專業書籍主要從微分幾何點觀點來推演地圖投影，旨在為製圖，地理資訊系統，空間資訊，計算機視覺，大地測量，地球物理和其他相關領域的學生和從事者提供完整地系統性之理論架構。

尤瑞哲

台南, 2015 年 5 月 6 日